Concurrent Games with Multiple Topologies

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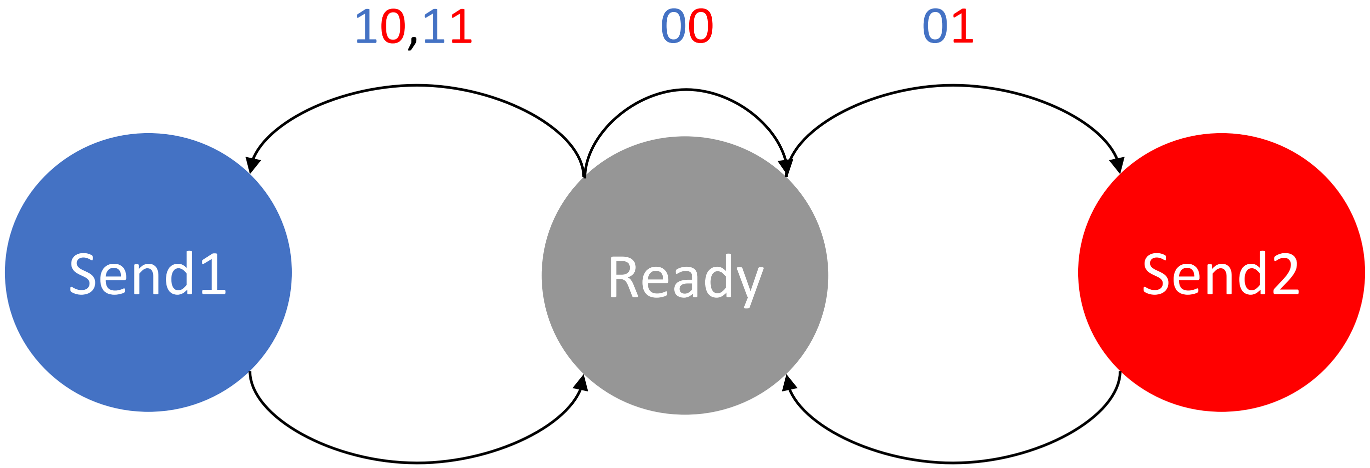
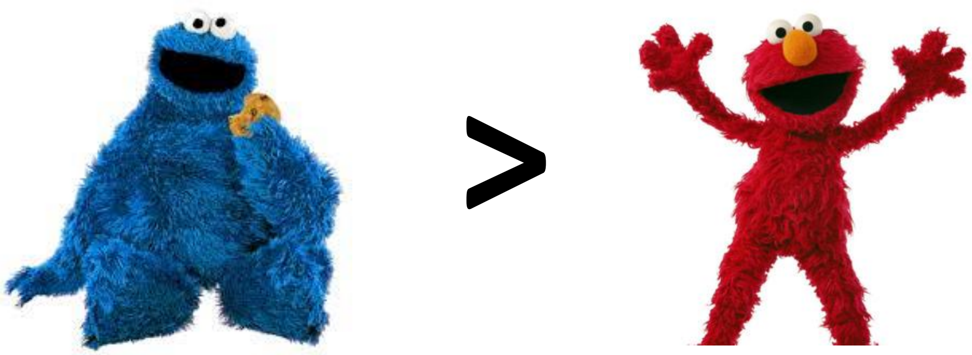
Concurrent Games

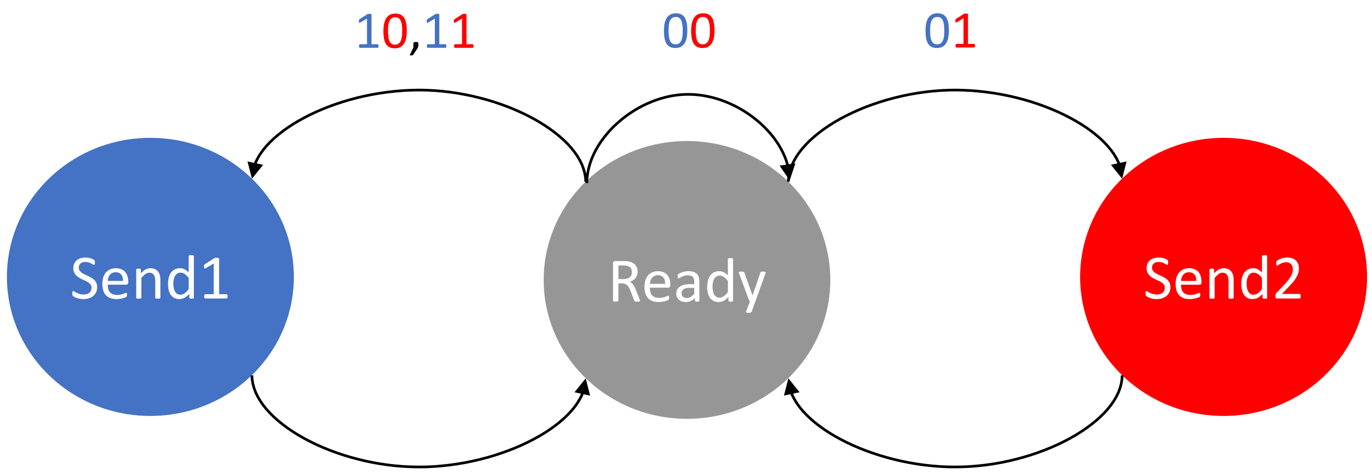
- States
- Players
- Actions
- Transitions
- Objectives





Concurrent Games – Strategies

- Strategy
  $\sigma_p\colon States^+ \rightarrow Act$
- Strategy Profile
- Outcome



Blue: if the two last states are *send1,ready* then take action 0 otherwise take action 1

Red: always take action 1

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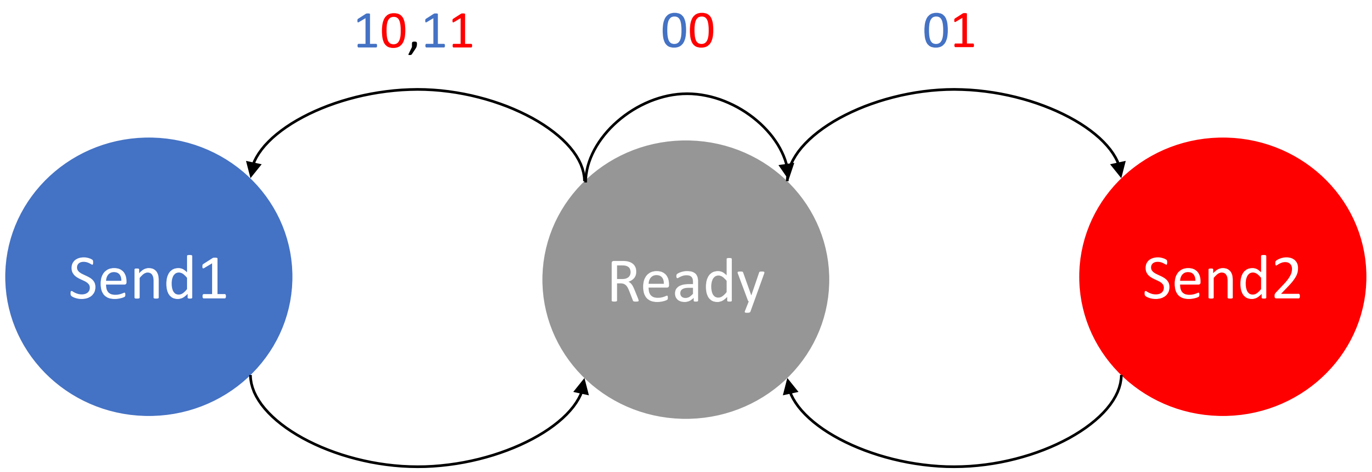
Solution Concept

Nash Equilibrium (NE): a strategy profile such that no player can unilaterally deviate and improve her outcome.

Blue: always take 1 Red: always take 1	Blue: alternate 1 and 0 Red: alternate 0 and 1

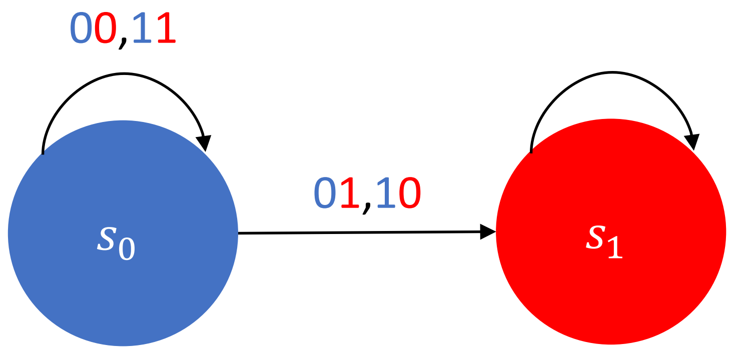


Solution Concept

Nash Equilibrium (NE): a strategy profile such that no player can unilaterally deviate and improve her outcome.

NE existence problem: Given a concurrent game G, does there exist a NE in G?

No NE!



Concurrent Games with Multiple Topologies

- What if players don't know their ports?
- Game Symmetry
- Multi-Topology Games (MTGs)

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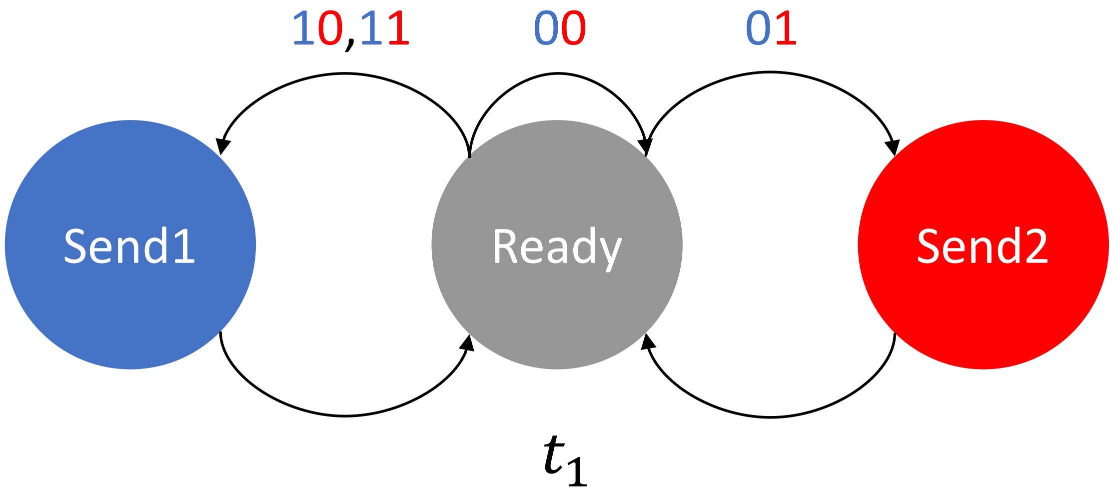
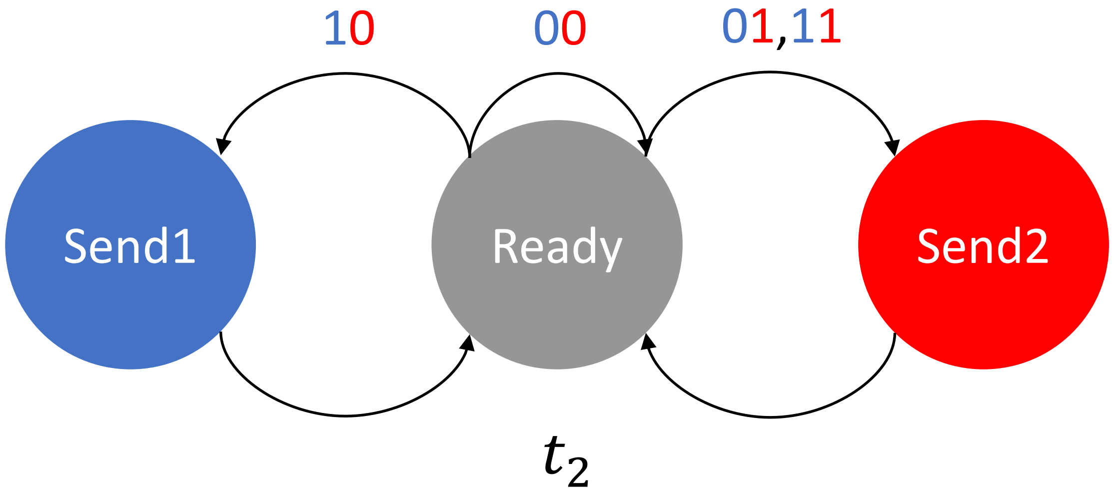
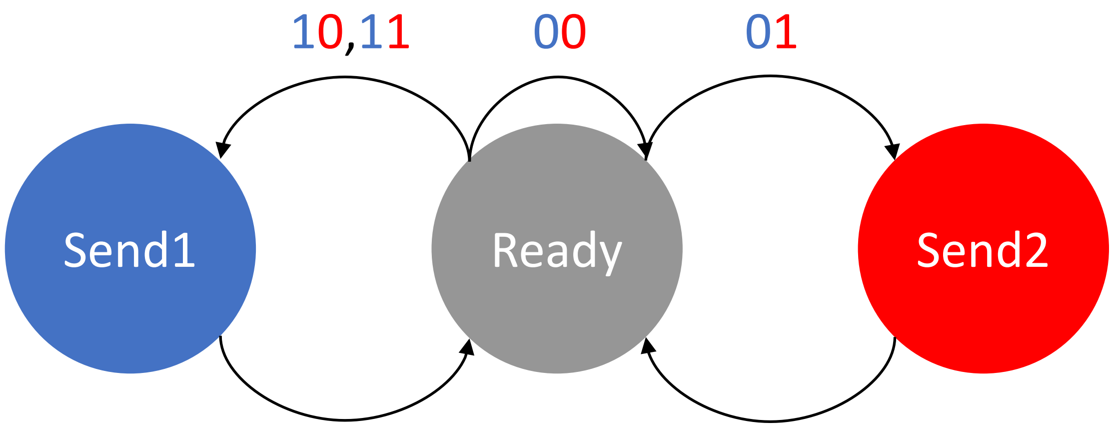


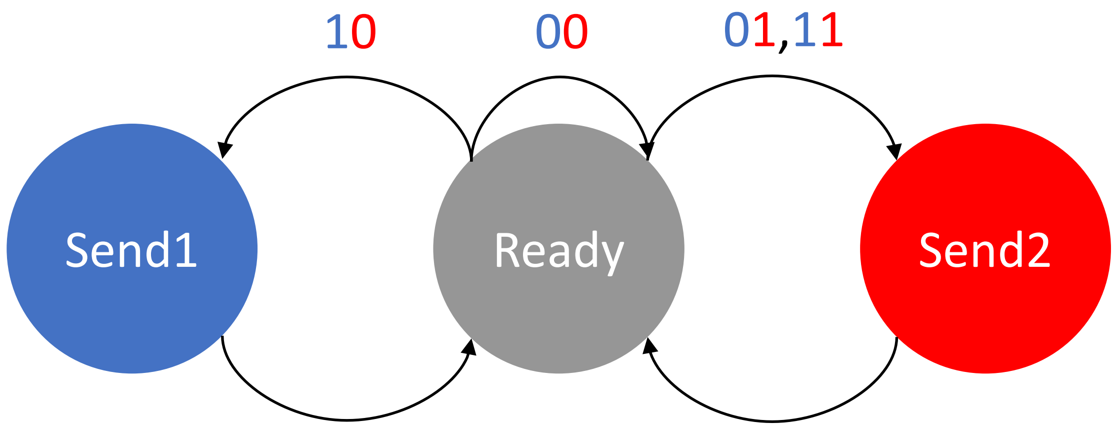


Multi-Topology Games (MTGs)

Blue: always take 1 **Red**: always take 1	**Blue**: alternate 1 and 0 **Red**: alternate 0 and 1
$Out_{t_1} = \text{Ready}, \text{Send1}, \text{Ready}, \text{Send1}, \ldots$ $Out_{t_2} = \text{Ready}, \text{Send2}, \text{Ready}, \text{Send2}, \ldots$ **Blue** wins $\{t_1\}$, **Red** wins $\{t_2\}$	$Out_{t_1} = \text{Ready}, \text{Send1}, \text{Ready}, \text{Send2}, \ldots$ $Out_{t_2} = \text{Ready}, \text{Send1}, \text{Ready}, \text{Send2}, \ldots$ **Blue** wins $\{t_1, t_2\}$, **Red** wins $\{t_1, t_2\}$



t_1



t_2

Solution Concept for MTGs

- Nash Equilibrium?

Nash Equilibrium (NE): a strategy profile such that no player can unilaterally deviate and *improve her outcome*.

- Multiple outcomes
- Deviation might affect topologies differently
- Idea – use the set of winning topologies

When a Player Might Deviate?

- **Greedy**: Better outcome in at least one topology.
- **Conservative**: Outcomes are at least as good in all topologies, and better in at least one.
- **Counting**: Increase the number of winning topologies.
- …

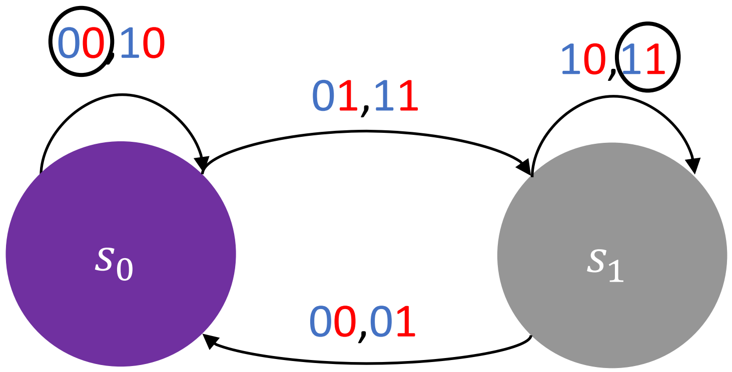
Conservative Nash Equilibrium

Conservative Nash Equilibrium (CNE): A strategy profile, such that no player can unilaterally deviate and win a topology that she is currently losing, without losing a topology that she is currently winning.

Blue = **Red**:
Last state is s_0 ➔ take action 0
Last state is s_1 ➔ take action 1

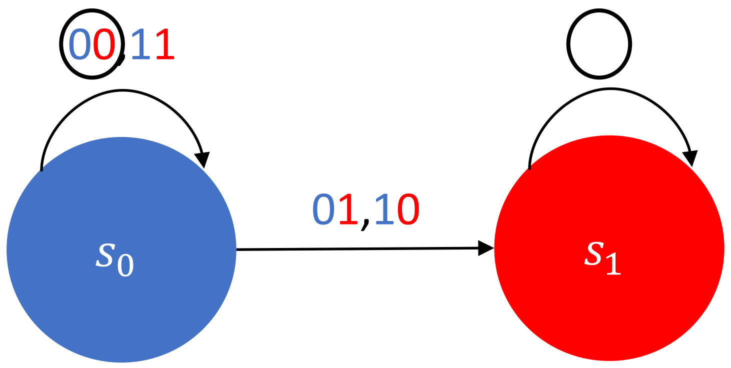
Blue wins $\{t_1, t_2\}$

Red wins $\{t_1\}$



t_1

CNE!



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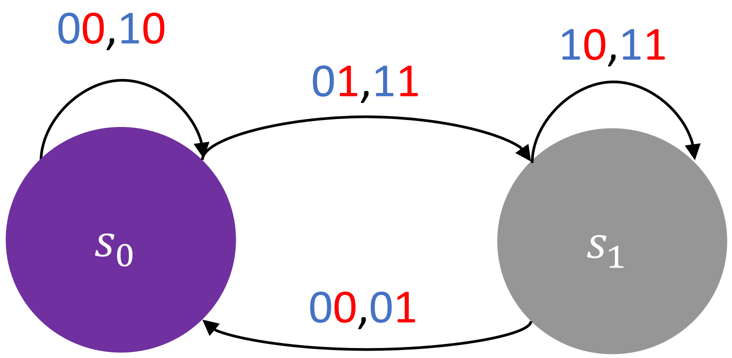
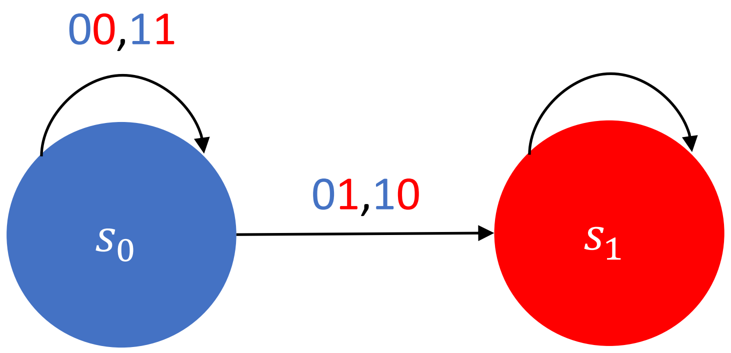
CNE Existence Problem

Conservative Nash Equilibrium (CNE): A strategy profile, such that no player can unilaterally deviate and win a topology that she is currently losing, without losing a topology that she is currently winning.

CNE existence problem: Given an MTG G, does there exist a CNE in G?

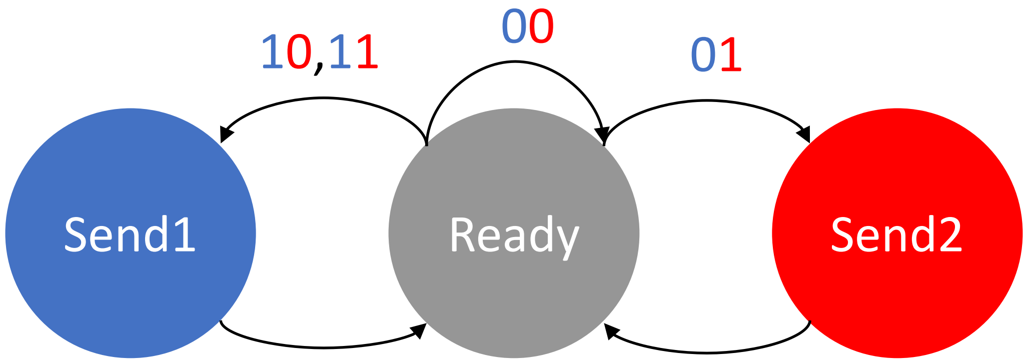


t_1

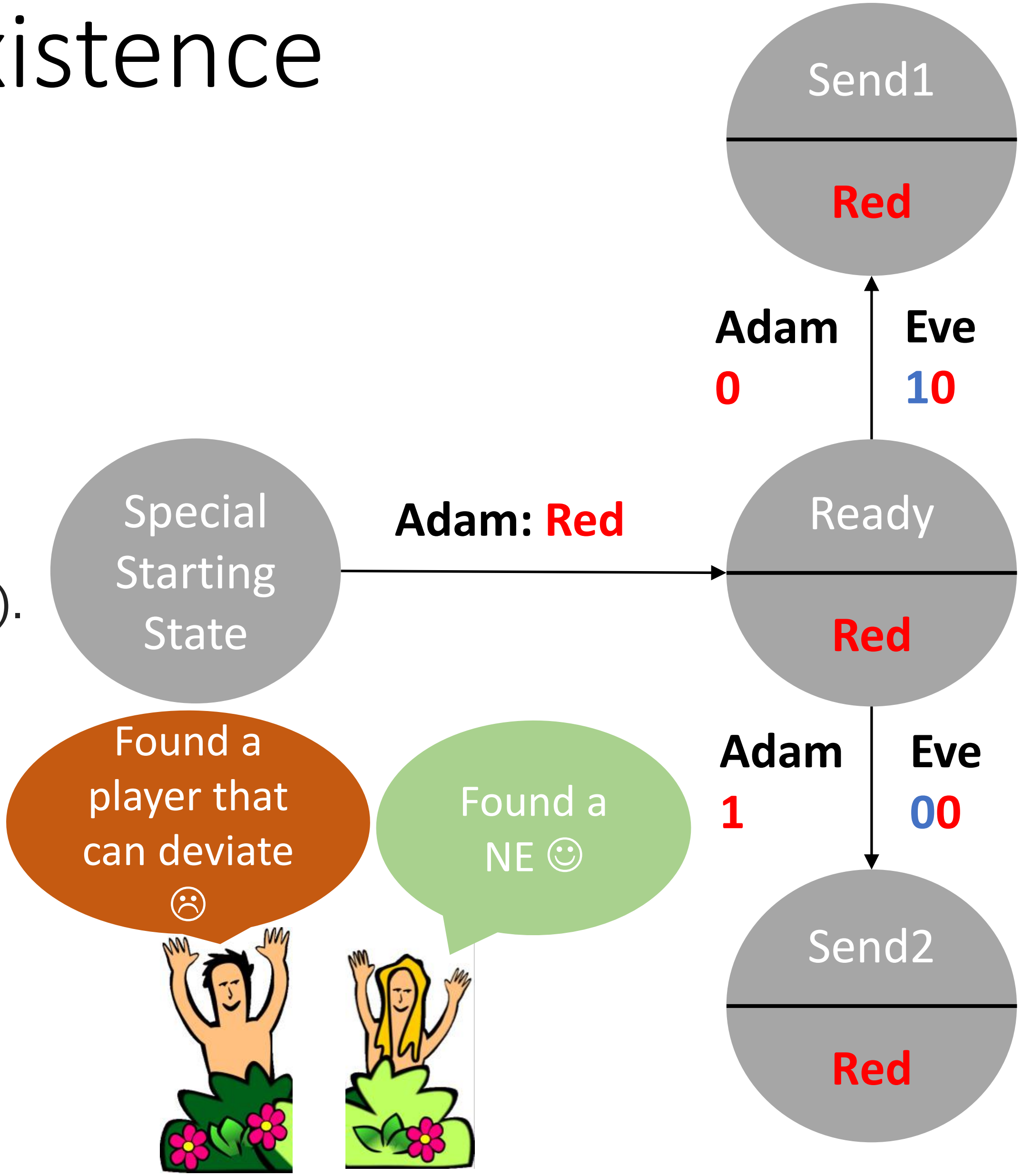


t_2

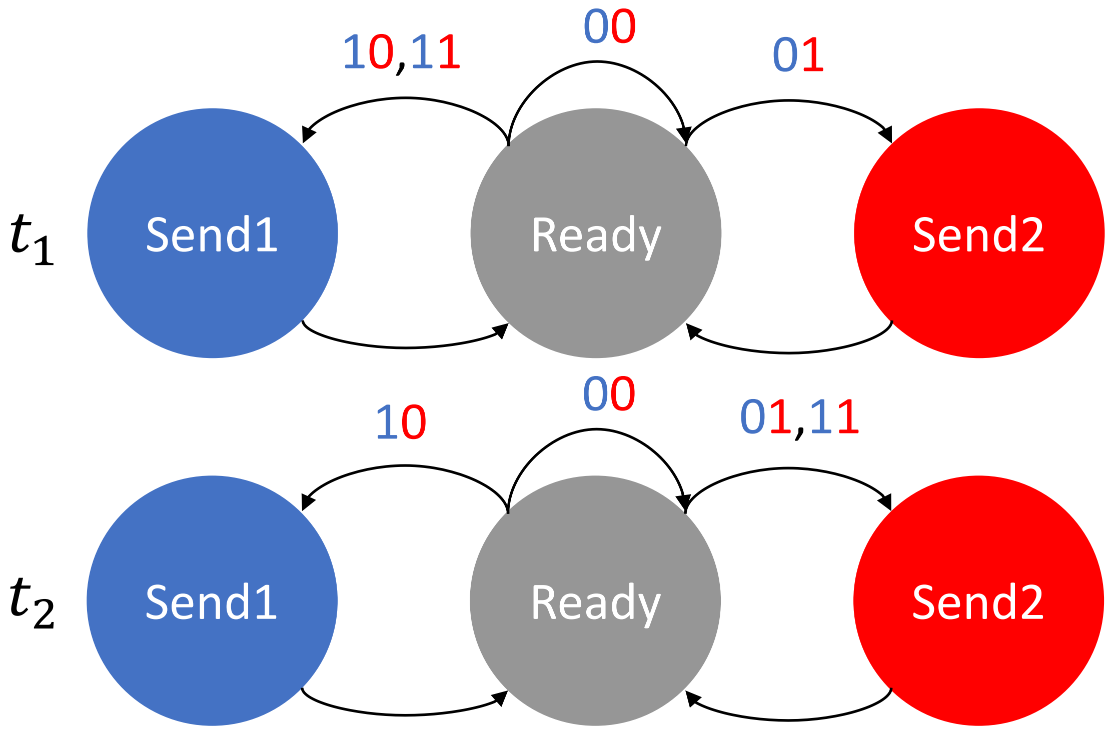
Suspect Game – NE existence

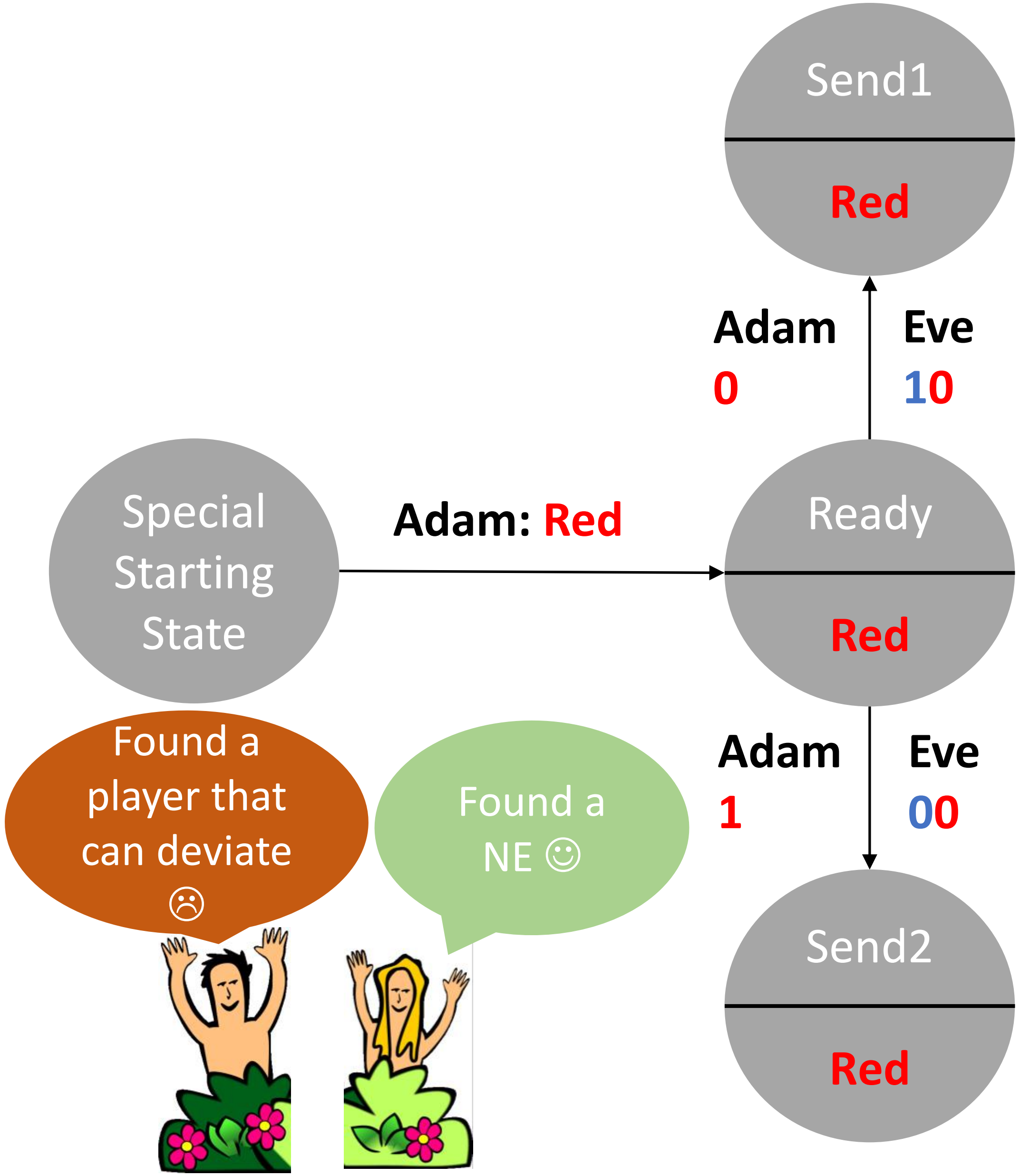


Bouyer, P. P., Brenguier, R., & Markey, N. N. (2015). Pure Nash Equilibria in Concurrent Games. *Logical methods in computer science*.



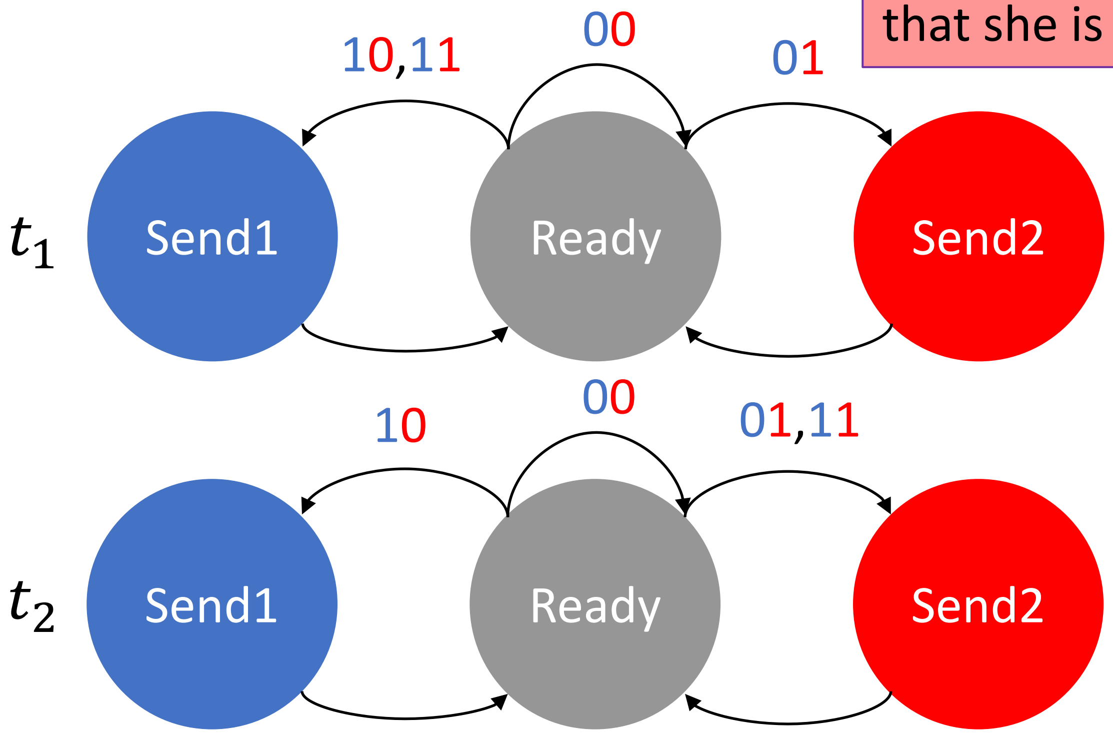
CNE Existence

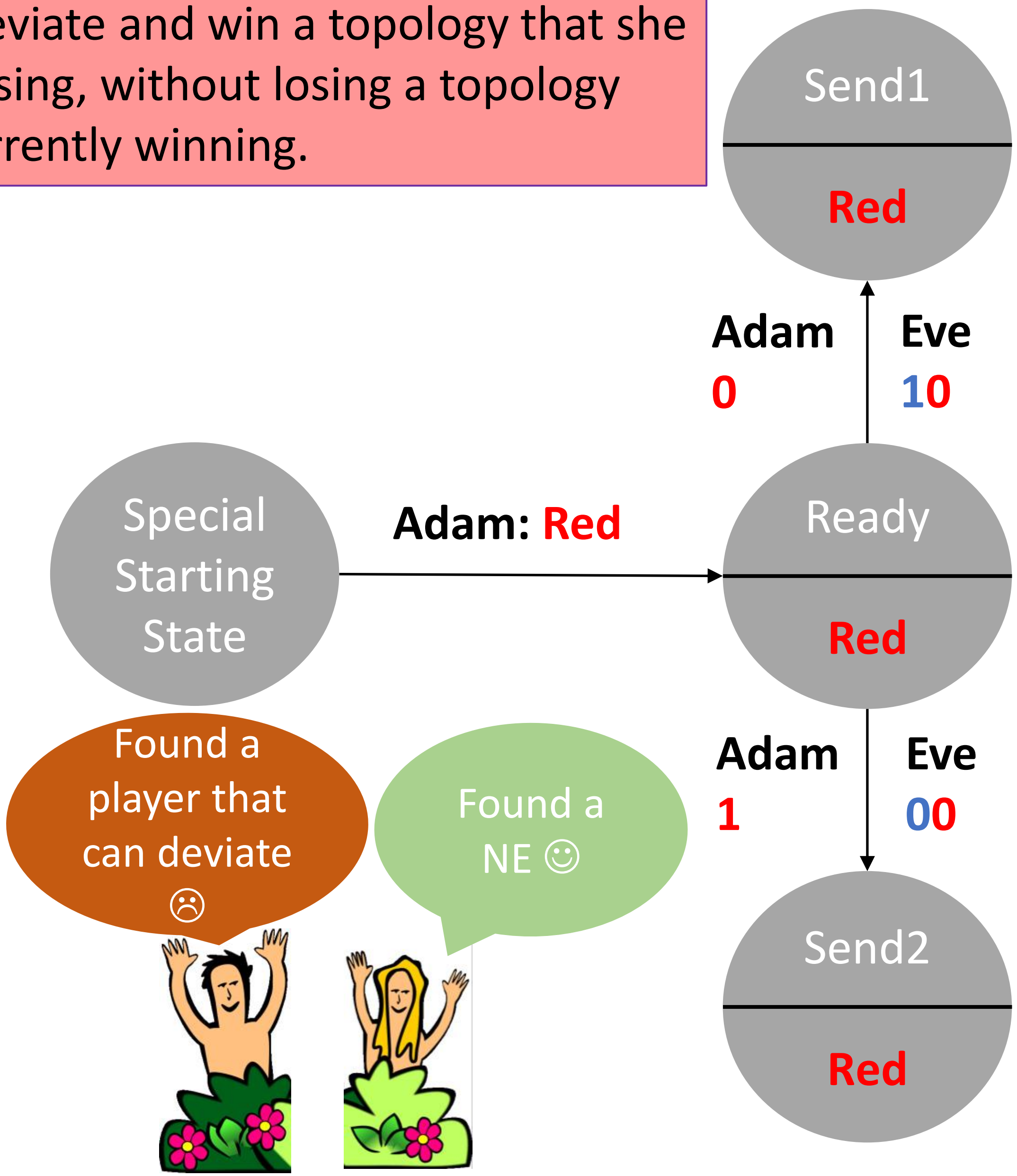




CNE Existence

CNE: A strategy profile, such that no player can unilaterally deviate and win a topology that she is currently losing, without losing a topology that she is currently winning.



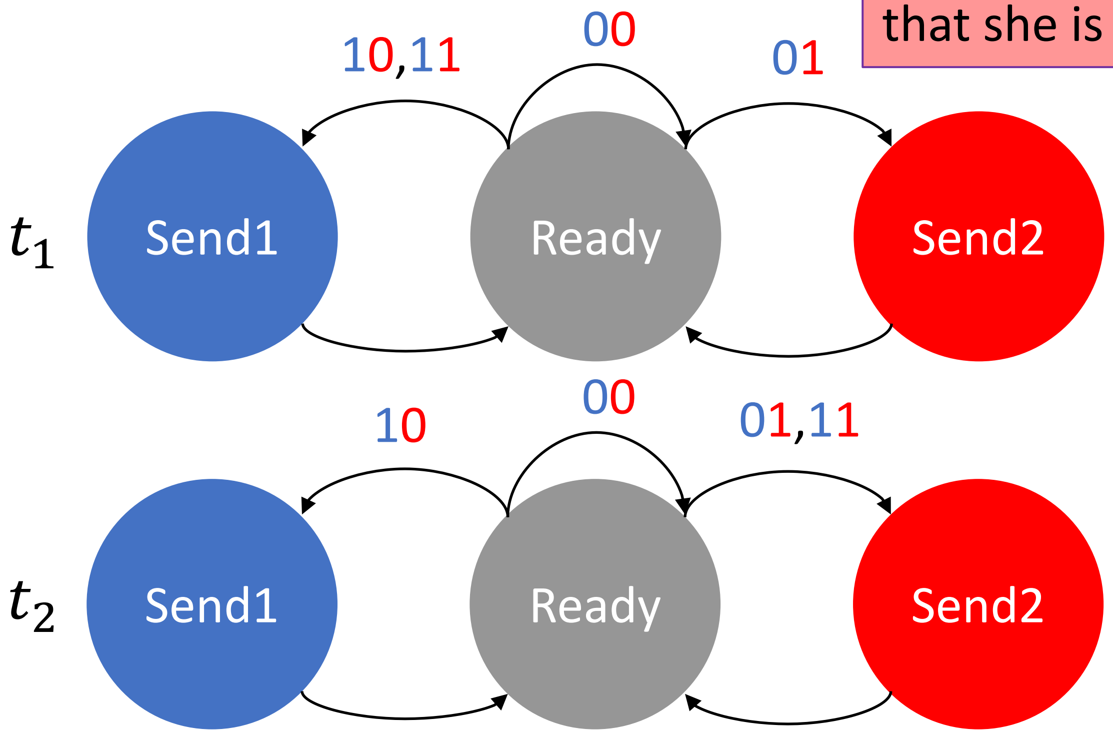


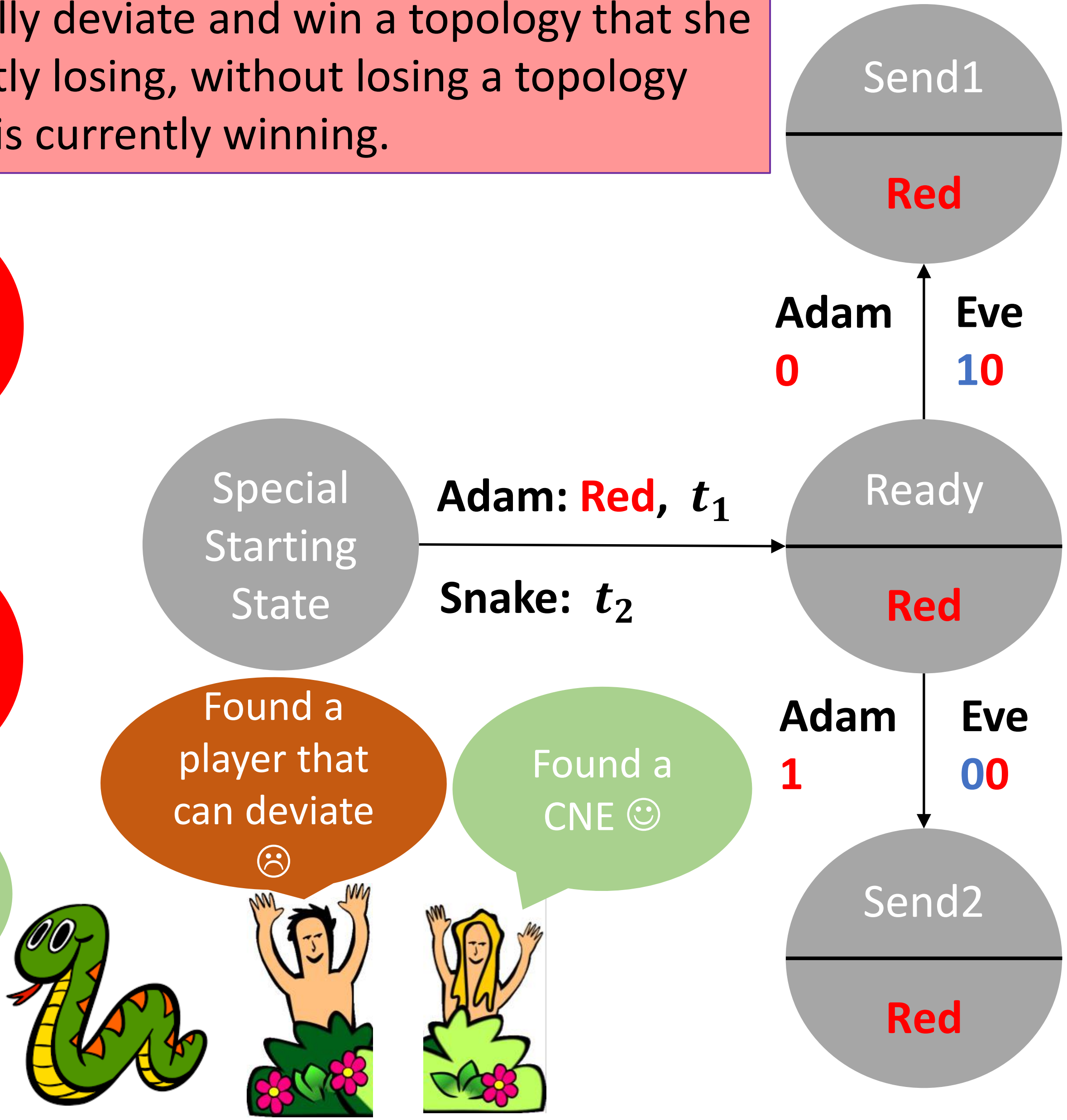
Guess

Set of winning topologies for each player

CNE Existence

CNE: A strategy profile, such that no player can unilaterally deviate and win a topology that she is currently losing, without losing a topology that she is currently winning.





The deviation doesn't work ☺

Found a player that can deviate ☹

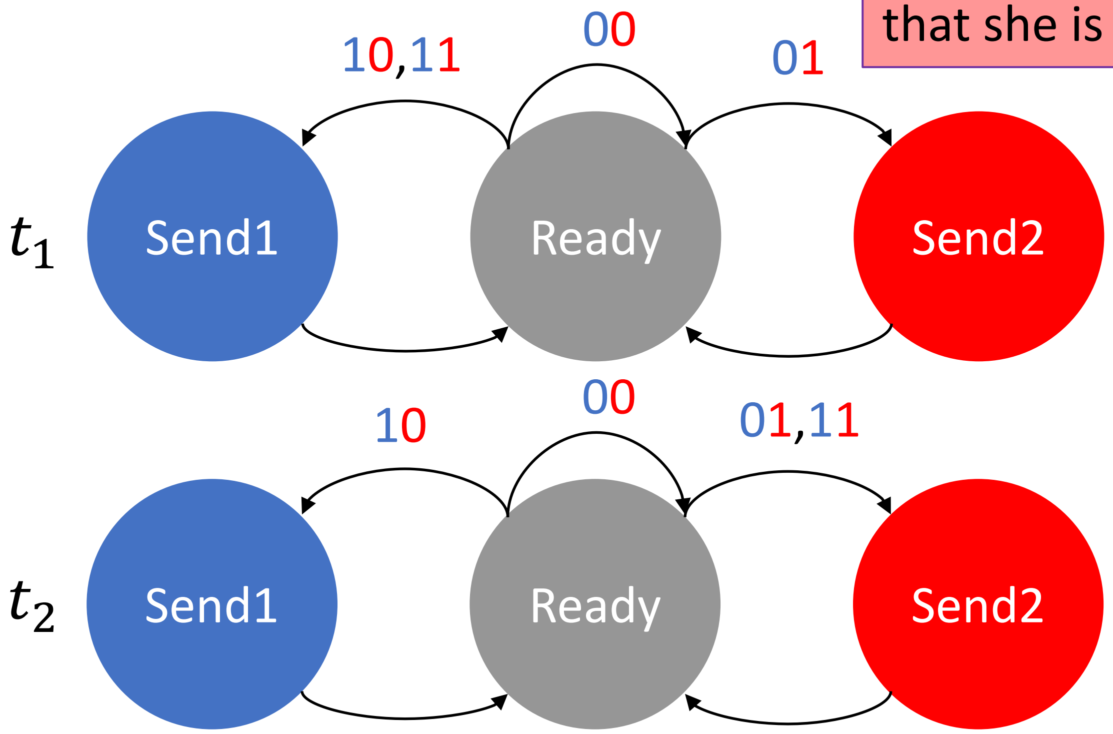
Found a CNE ☺

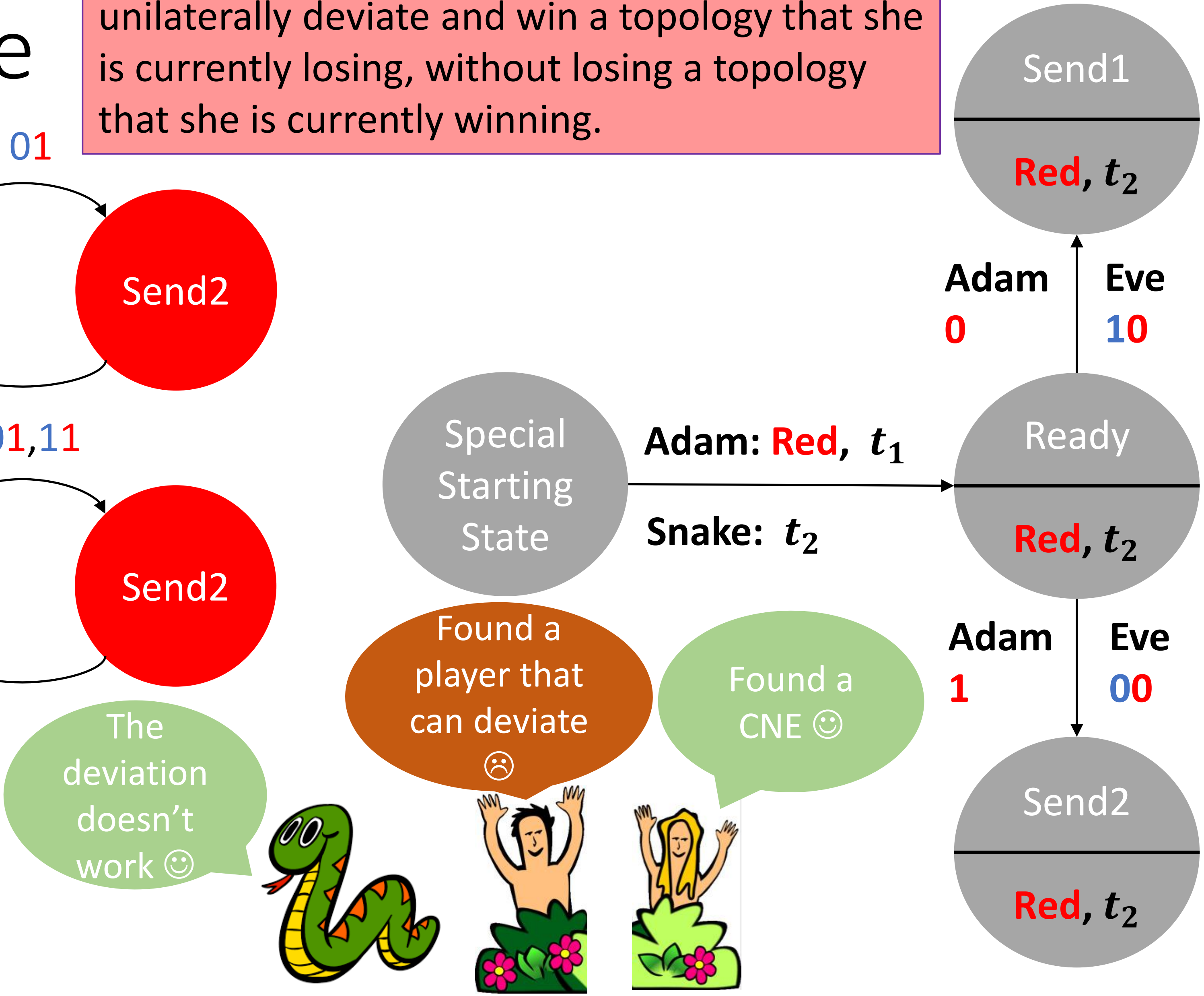
Guess

Set of winning topologies for each player

CNE Existence

CNE: A strategy profile, such that no player can unilaterally deviate and win a topology that she is currently losing, without losing a topology that she is currently winning.

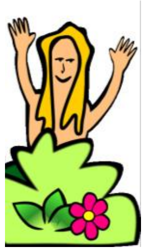




Guess

Set of winning topologies for each player

Three-Player Partial-Information Games

$\exists$  $\forall$  $\exists$  φ

- Undecidable in the general case

- If **Eve** is *less informed* than **Adam** the problem is 2-EXPTIME complete

Chatterjee, K., & Doyen, L. (2014). Games with a weak adversary. *International Colloquium on Automata, Languages, and Programming.*

Contributions

- Multi-Topology Games
- Reduction to three-player partial-information games
- Border of decidability, MTGs as restricted form of partial-information

Thanks for Listening!

Questions?